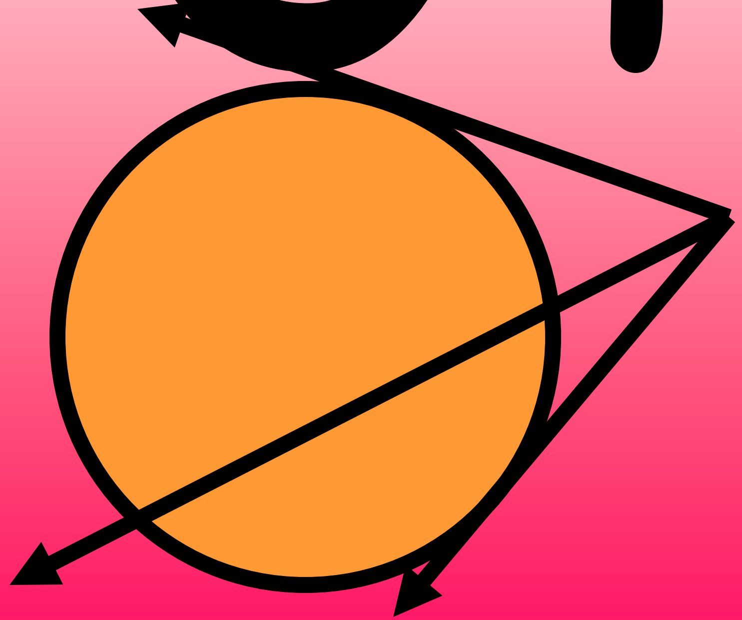
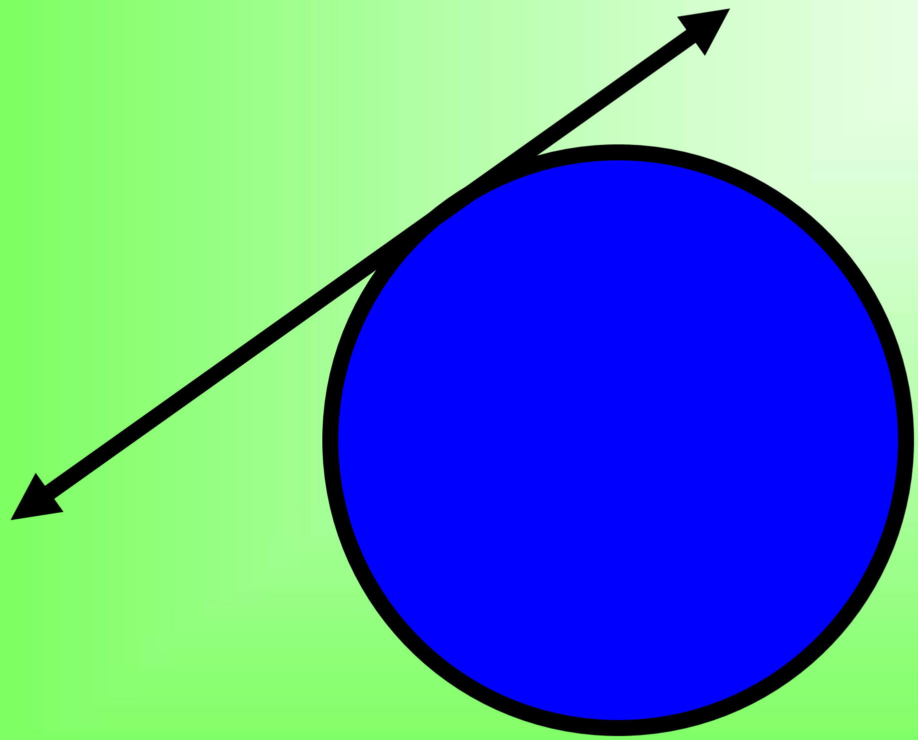
Section 9.6

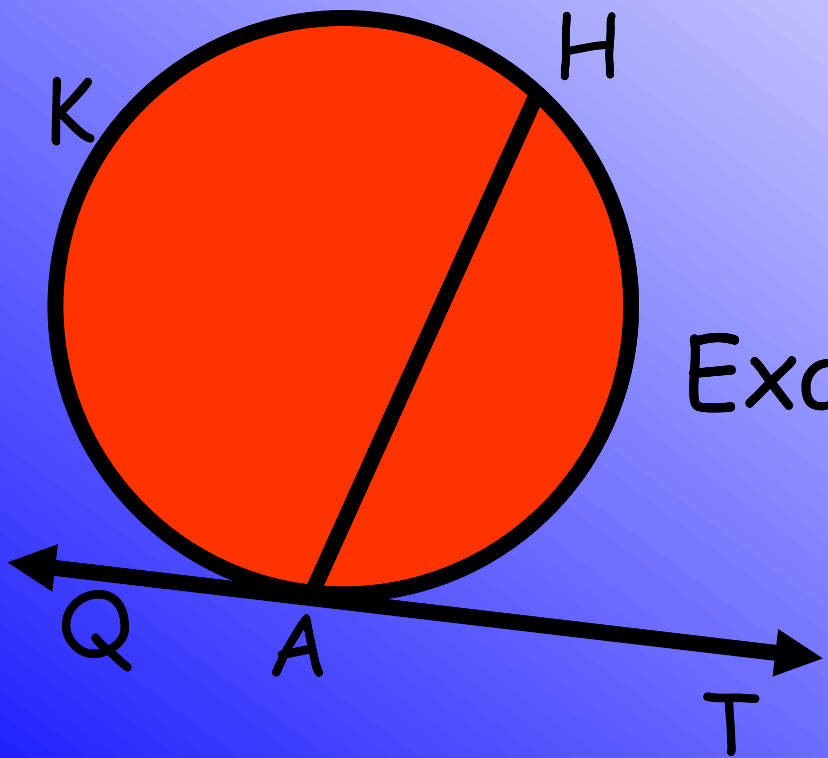
Other Angles



Recall – a tangent is a line that intersects a circle exactly one time.



Theorem 9-8: <u>The measure of an angle formed by a chord and a tangent is equal to half the measure of the intercepted arc.</u>

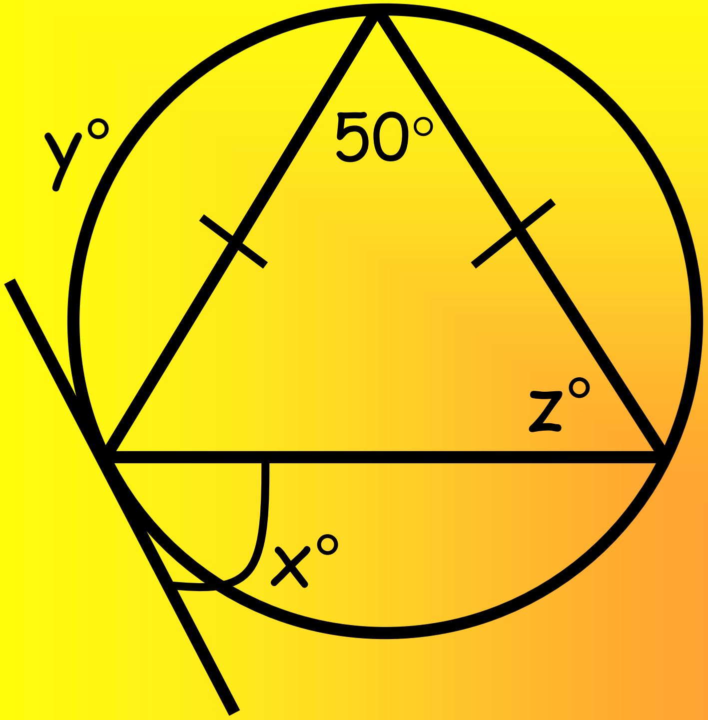


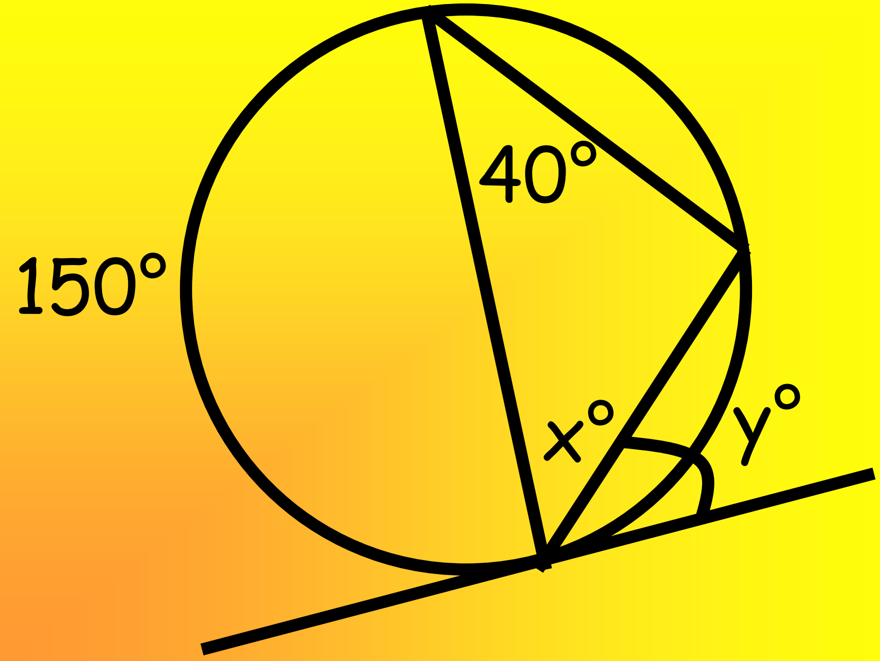
$m\angle HAT = \frac{1}{2} m\overset{\frown}{HA}$.

Example: If $m\angle HAT = 75$, then $m\overset{\frown}{HA} = 150$

And… $m\angle HAQ = 105$, then $m\overset{\frown}{HKA} = 210$

Examples





x = 50°

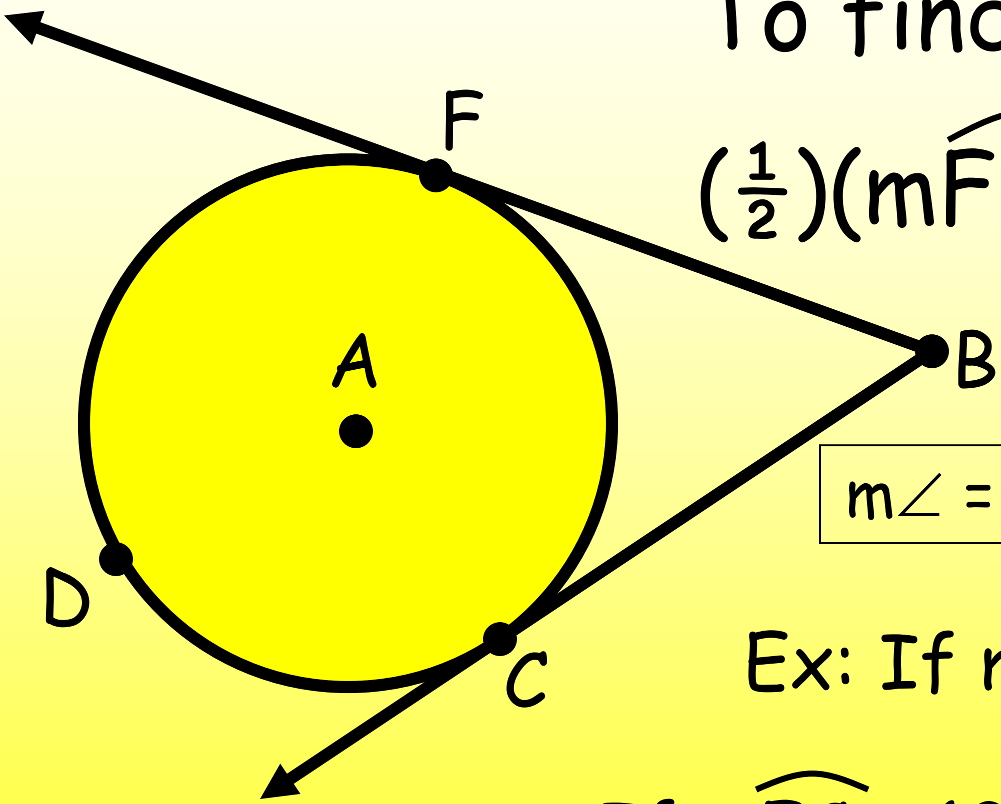
y = 130°

z = 65°

x = 65°

y = 40°

Case 1 – Two tangents that form an angle outside of a circle.



To find the m∠FBC…

$(\frac{1}{2})(m\overset{\frown}{FDC} - m\overset{\frown}{FC}) = m\angle FBC$

m∠ = ½ (Bigger Arc – Smaller Arc)

Ex: If $m\overset{\frown}{FC} = 120$, find m∠FBC.

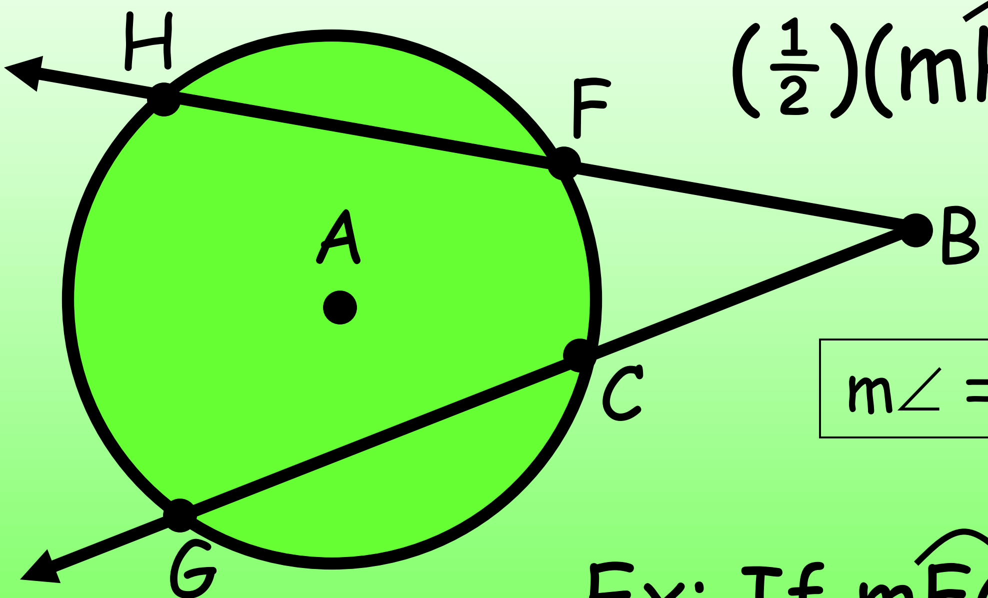
If $m\overset{\frown}{FC} = 120$, then $m\overset{\frown}{FDC}$ = 240.

$m\angle FBC = (\frac{1}{2})(240 - 120) = 60$

Case 2 – Two secants that form an angle outside of a circle.

To find the $m\angle FBC$...

$$\left(\tfrac{1}{2}\right)(m\overset{\frown}{HG} - m\overset{\frown}{FC}) = m\angle FBC$$



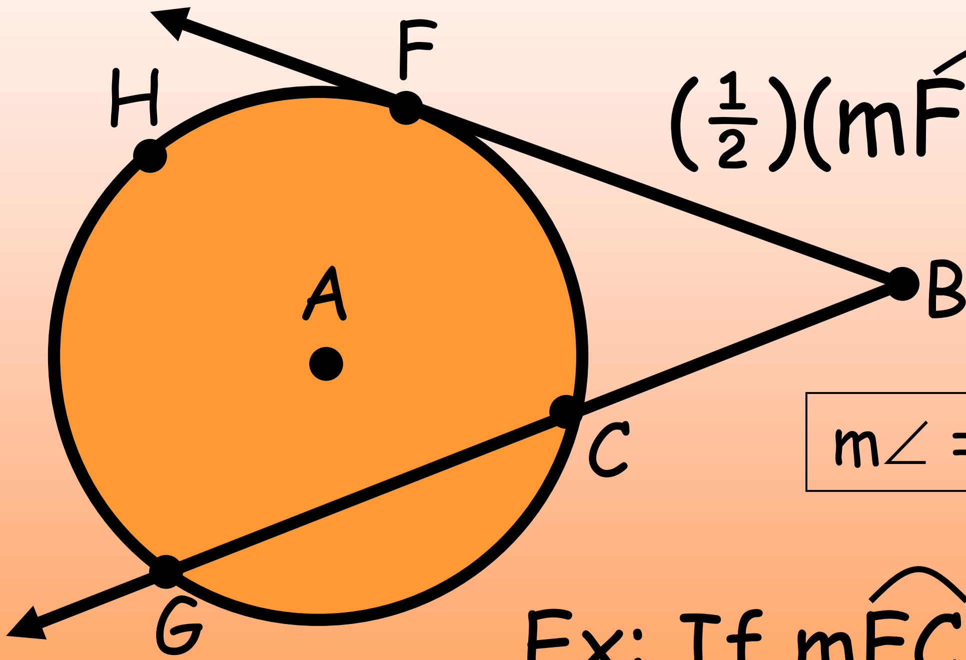
$m\angle = \frac{1}{2}$ (Bigger Arc – Smaller Arc)

Ex: If $m\overset{\frown}{FC} = 80$, and $m\overset{\frown}{HG} = 150$ find $m\angle FBC$.

$m\angle FBC = \underline{\left(\tfrac{1}{2}\right)(150 - 80) =}\ \underline{35}$

Case 3 – A secant and a tangent form an angle outside of a circle.

To find the $m\angle FBC$...



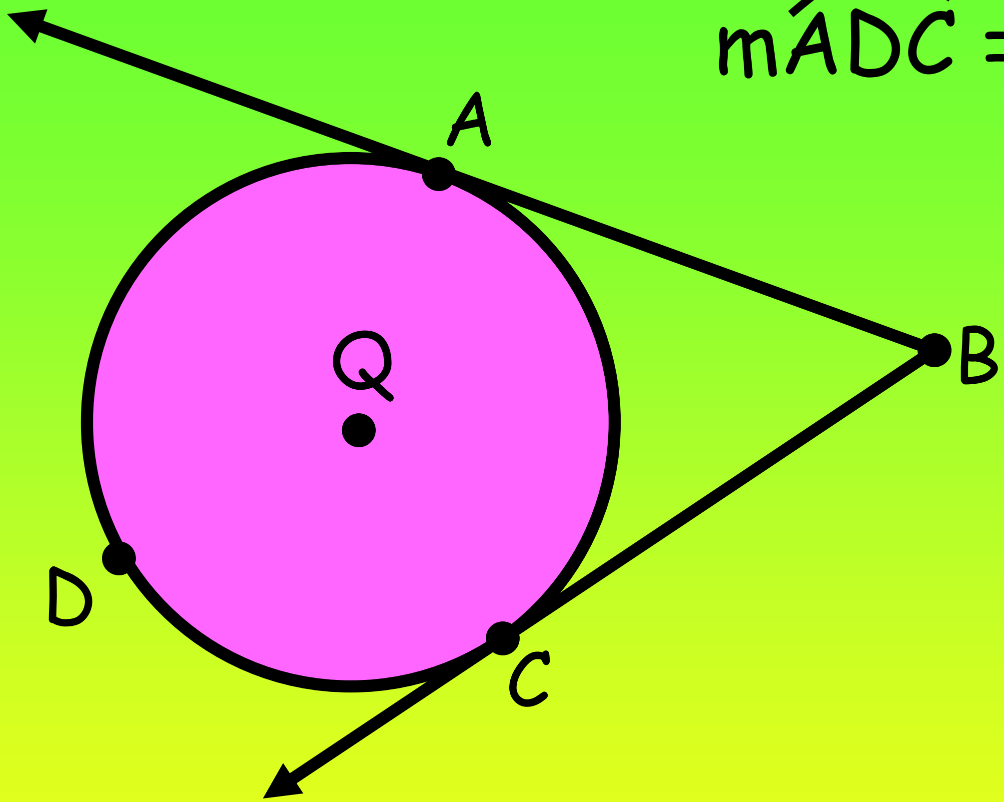
$\left(\frac{1}{2}\right)(m\overset{\frown}{FHG} - m\overset{\frown}{FC}) = m\angle FBC$

$m\angle = \frac{1}{2}$ (Bigger Arc – Smaller Arc)

Ex: If $m\overset{\frown}{FC} = 80$, and $m\overset{\frown}{FHG} = 160$ find $m\angle FBC$.

$m\angle FBC = \underline{\left(\frac{1}{2}\right)(160 - 80) =} \underline{40^\circ}$

Example 1



$m\overset{\frown}{ADC} = 222$. Find $m\angle ABC$

$m\overset{\frown}{AC} = 138$

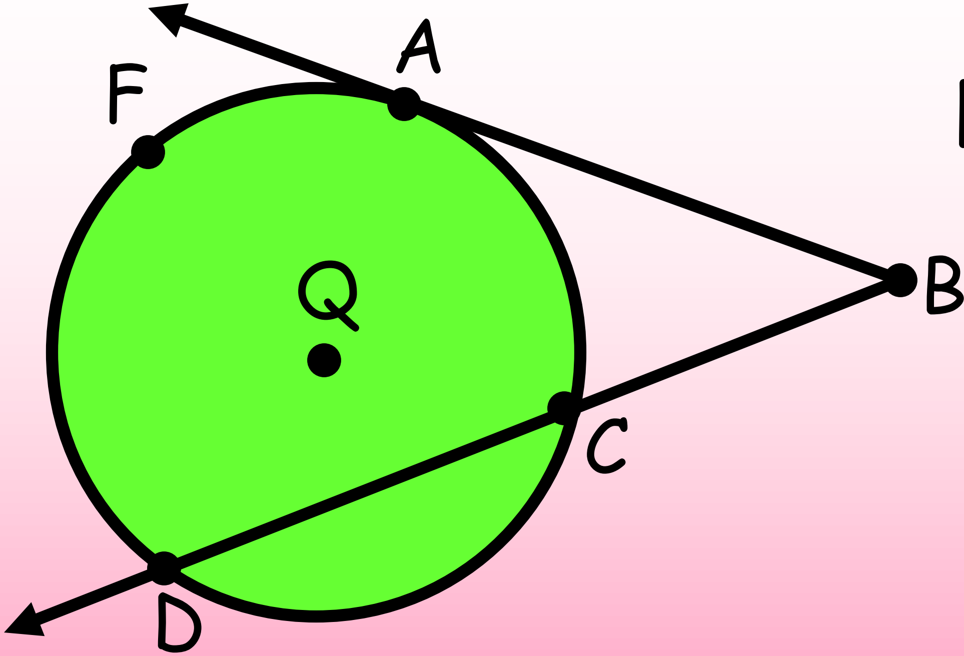
$\left(\frac{1}{2}\right)(m\overset{\frown}{ADC} - m\overset{\frown}{AC}) = m\angle ABC$

$\frac{1}{2}(222 - 138) = \frac{1}{2}(84)$

$m\angle ABC = 42$

Example 2

$m\overset{\frown}{AFD} = 175$ and $m\overset{\frown}{DC} = 70$.



Find $m\angle ABC$

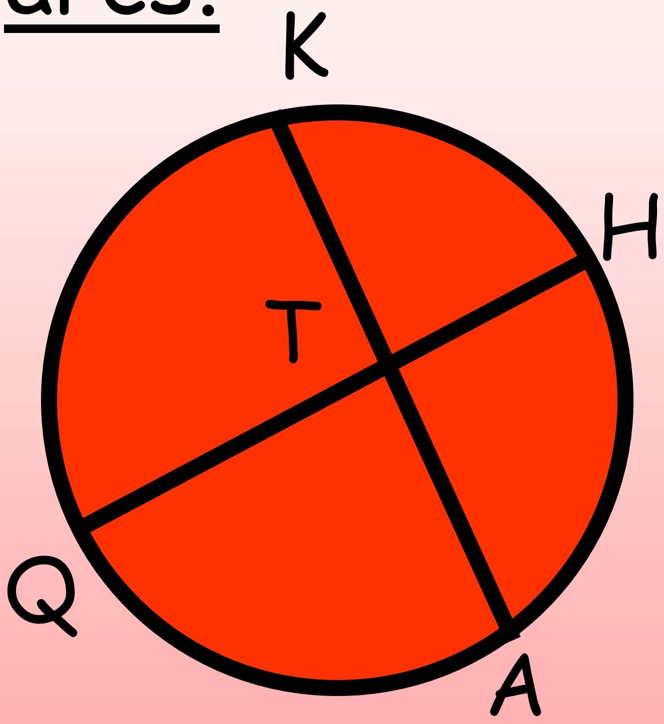
$m\overset{\frown}{AC} = 115$

$(\frac{1}{2})(m\overset{\frown}{AFD} - m\overset{\frown}{AC}) = m\angle ABC$

$\frac{1}{2}(175 - 115) = \frac{1}{2}(60)$

$m\angle ABC = 30$

Theorem 9-9: The measure of an angle formed by two chords is equal to half the sum of the measures of the intercepted arcs.



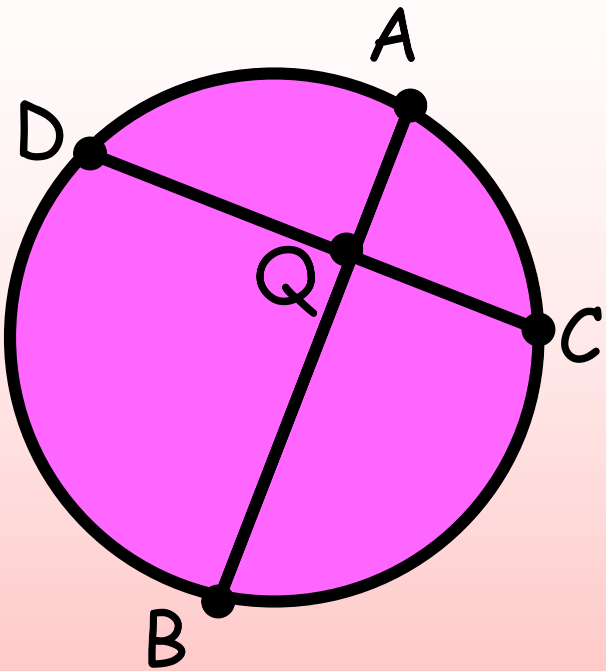
$m\angle QTA = \frac{1}{2}(m\overset{\frown}{QA} + m\overset{\frown}{KH})$.

$m\angle = \frac{1}{2}$ (Bigger Arc + Smaller Arc)

Example: If $m\overset{\frown}{QA} = 50$, $m\overset{\frown}{KH} = 70$,

then $m\angle QTA = 60$

Example 1



$m\overset{\frown}{AC} = 55$ and $m\overset{\frown}{DB} = 145$.

Find $m\angle DQB$.

$$\left(\tfrac{1}{2}\right)(m\overset{\frown}{DB} + m\overset{\frown}{AC}) = m\angle DQB$$

$$\tfrac{1}{2}(145 + 55) = \tfrac{1}{2}(200)$$

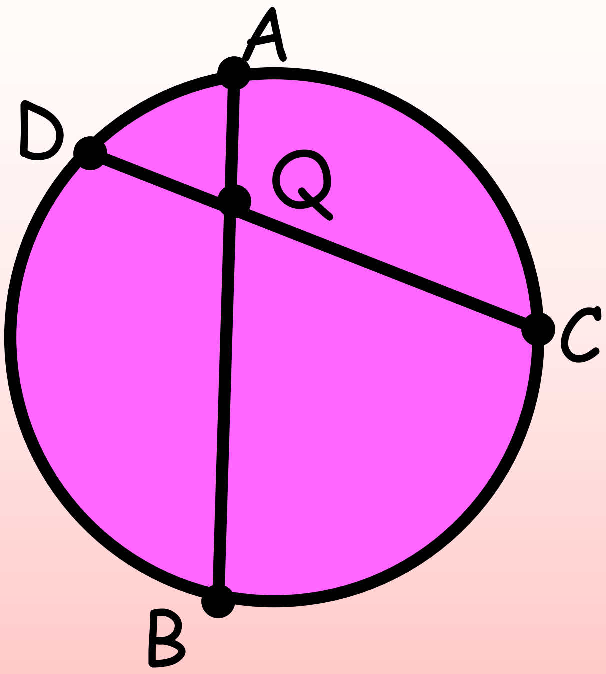
$$m\angle DQB = 100$$

$$m\angle AQC = 100$$

Example 2

$m\overset{\frown}{AD} = 15$ and $m\angle DQA = 75.$

Find $\overset{\frown}{BC}$.



$$\left(\tfrac{1}{2}\right)(m\overset{\frown}{BC} + m\overset{\frown}{DA}) = m\angle DQA$$

$$\tfrac{1}{2}(x + 15) = 75$$

$$x + 15 = 150$$

$$m\overset{\frown}{BC} = 135$$

$$m\angle AQC = 105$$